ON GENERALISED MHD COUETTE FLOW IN SLIP-FLOW REGIME

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ABSTRACT. An analysis of combined influence of electric and magnetic fields on generalised Couette flow of an electrically conducting, incompressible, viscus fluid is carried out. Closed form solutions are obtained for velocity, current density and flow rate under first order velocity slip conditions. They are shown on graphs. The numerical values of the velocity gradient are entered in a table. The results are discussed in the conclusion.

NOMENCLATURE

B_0 = Magnetic field

b_x = Component of Magnetic field in x-direction

E_y = Electric field componenent in the y-direction

f = Non-dimensional velocity component in x-direction

h = Separation between the plates

j_y = Current density

K = Non-dimensional electric field parameter

M = Hartmann number

p = Pressure

P = Non-dimensional pressure

P_x = Non-dimensional pressure gradient in x-direction

P_r = Prandtl number

Q = Flow rate

R_e = Reynolds number

R = gas constant

T = Temperature

u = Velocity component in x -direction

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x, y, z = co-ordinates

σ = electrical conductivity of the fluid

μ = viscosity of the fluid

λ = rarefaction parameter

ξ, η = non-dimensional co-ordinates in x and z directions

τ = shearing stress

ϵ_{1u} = the slip coefficient

$= \left(\frac{2-\beta}{\beta}\right) \bar{l}$, β is Maxwell's reflection coefficient

$\bar{l}$ = mean free path, $(\sqrt{\pi/8}/0.499)\, \frac{\mu\sqrt{RT}}{P_r}$

μ_0 = magnetic permeability.

INTRODUCTION

Generalised *mhd* Couette flow of an electrically conducting incompressible, viscous fluid was studied by Agarwal (1962) and Soundalgeker (1967). Soundalgekar studied it under the action of both the electric and magnetic fields. A number of researchers have studied plane Couette flow with or without magnetic field the reference to which are mentioned in the paper by Soundalgekar (1967).

But when the fluid is flowing at high temperature, the density of the fluid is slightly reduced and hence to describe such a flow of rarefied gas the usual no-slip boundary conditions are replaced by the first-order velocity slip boundary conditions with the modified Navier-Stokes equations describing the flow field. Such an attempt in case of channel flow was made recently by Inman (1965).

The object of the present paper is to study the problem considered by Soundalgekar (1967) under first order velocity slip boundary conditions and to bring out the effects of rarefaction on the flow field. Hence in section 2, the repeated problem is solved completely and closed form solutions are obtained for the velocity, current density, velocity gradient at the upper moving wall, shearing stress at the lower stationary wall and mass flow. The numerical values for velocity gradient are entered in the table, whereas others are shown on graphs. In section 3, the conclusions are set out.

MATHEMATICAL ANALYSIS

The steady flow of an incompressible, viscous electrically conducting fluid is assumed to be between two infinite parallel plates in the x and y directions, where X-axis is chosen along the lower stationary plate and Z-axis is taken normal to it. Let U be the uniform velocity of the upper plate in its own plane.

Then the equations governing the flow are (Soundalgekar 1967)

$$\frac{\partial p}{\partial x} = j_y B_0 + \mu \frac{d^2u}{dz^2} \tag{1}$$

$$\frac{dp}{dz} = -j_y b_x \tag{2}$$

where j_y, B_0, b_x etc. are defined in nomenclature.
From Ohm's law, we get

$$j_y = \sigma(E_y - uB_0) \tag{3}$$

The boundary conditions are (Dix, 1963)

$$\left.\begin{aligned} u &= \xi_u \left(\frac{du}{dz}\right) \text{ at } z = 0 \\ u &= U - \xi_u \left(\frac{du}{dz}\right) \text{ at } z = h \end{aligned}\right\} \tag{4}$$

where h is the separation between the two plates.

Eliminating j_y between (1) and (3) we get,

$$\frac{d^2u}{dz^2} + \sigma(E_y - uB_0)B_0 = \frac{\partial p}{\partial x} \tag{5}$$

Equation (5) now reduces to the following non-dimensional form

$$\frac{d^2f}{d\eta^2} - M^2 f = RP_x - M^2K \tag{6}$$

where

$$\begin{aligned} f &= \frac{u}{U}, \qquad \xi = \frac{x}{h}, \qquad \lambda = \xi_u/h \\ \eta &= \frac{z}{h} \qquad P = \frac{p}{\rho U^2} \\ K &= \frac{E_y}{UB_0} \qquad M^2 = \frac{\sigma B_0{}^2 h^2}{\mu} \\ R_e &= \frac{\rho U h^0}{\mu} \qquad P_x = \frac{\partial P}{\partial \xi} \end{aligned} \tag{7}$$

and (4) reduce to

$$\left.\begin{aligned} f &= \lambda \left(\frac{df}{d\eta}\right) \quad \text{at } \eta = 0 \\ f &= 1-\lambda \left(\frac{df}{d\eta}\right) \quad \text{at } \eta = 1 \end{aligned}\right\} \quad \ldots \quad (8)$$

where λ is the rarefaction parameter.

The solution of equation (6) satisfying the conditions (8) is

$$f = \frac{\sinh M\eta + \lambda M \cosh M\eta}{(1+\lambda^2 M^2)\sinh M + 2\lambda M \cosh M}$$

$$+\frac{RP_x - M^2K}{M^2}\left[\frac{\sinh M\eta + \sinh M(1-\eta) + \lambda M\{\cosh M\eta + \cosh M(1-\eta)\}}{(1+\lambda^2 M^2)\sinh M + 2\lambda M \cosh M}\right] \ldots \quad (9)$$

For $\lambda = 0$, this reduces to equation (19) of Soundalgekar (1967)

From (3), (7) and (9), we get

$$J_y = K \quad f \tag{10}$$

where

$$J_y = \frac{j_y}{\sigma U B_0}$$

The magnetic field can be determined from the relation.

$$j_y = \frac{1}{\mu_e} \quad \frac{db_x}{dz} \tag{11}$$

The effect of rarefaction on the distribution of the velocity profiles and current distribution is shown in figures 1 to 8 for different values of RP_x, M, K and λ.

The velocity gradient at the upper plate is given by

$$\left.\frac{df}{d\eta}\right|_{\eta=1} = \frac{M(\cosh M + \lambda M \sinh M}{(1+\lambda^2 M^2)\sinh M + 2\lambda M \cosh M} + \frac{RP_x - M^2K}{M}$$

$$\left[\frac{\cosh M - 1 + \lambda M \sinh M}{(1+\lambda^2 M^2)\sinh M + 2\lambda M \cosh M}.\right. \tag{12}$$

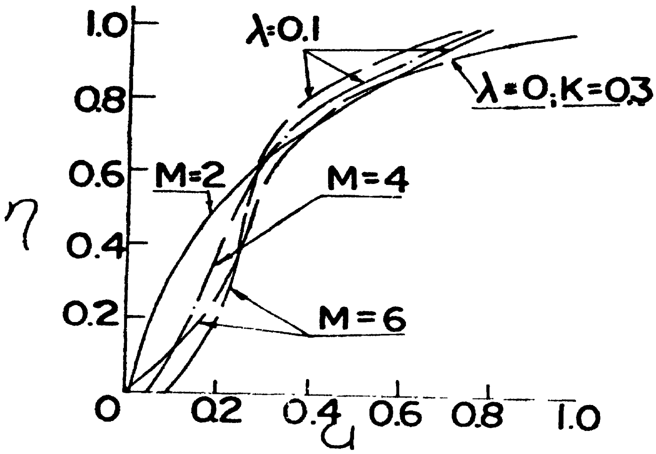


Figure 1.

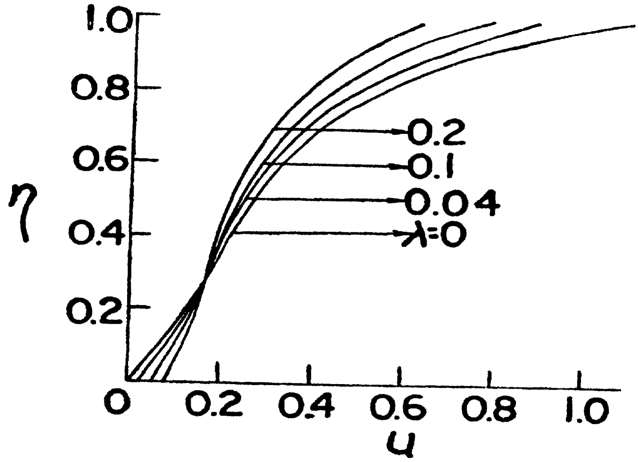
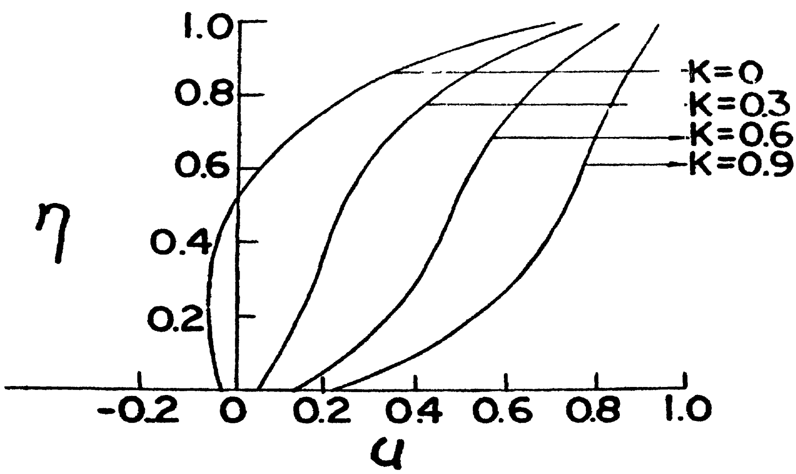


Figure 2.



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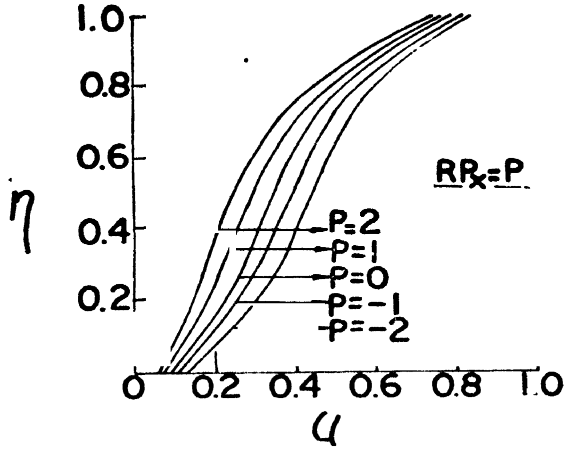

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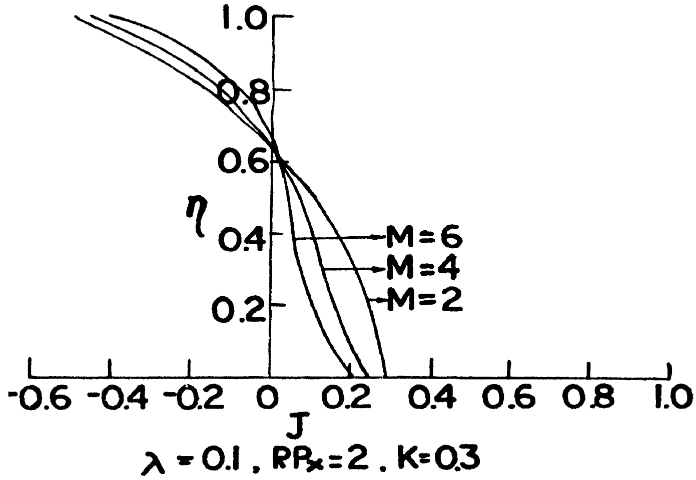

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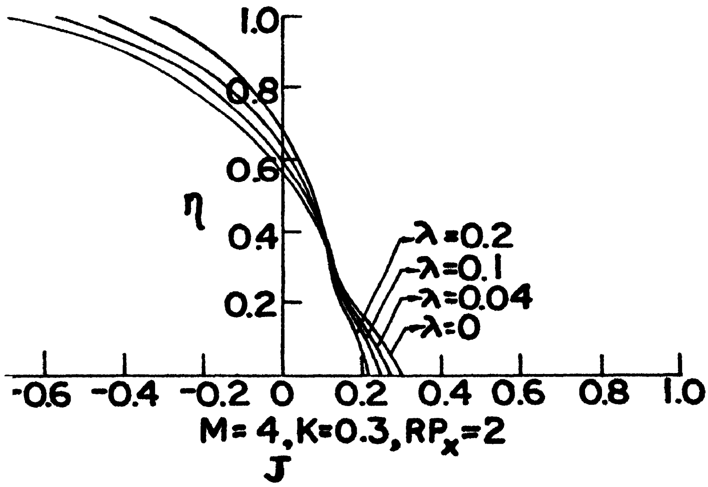

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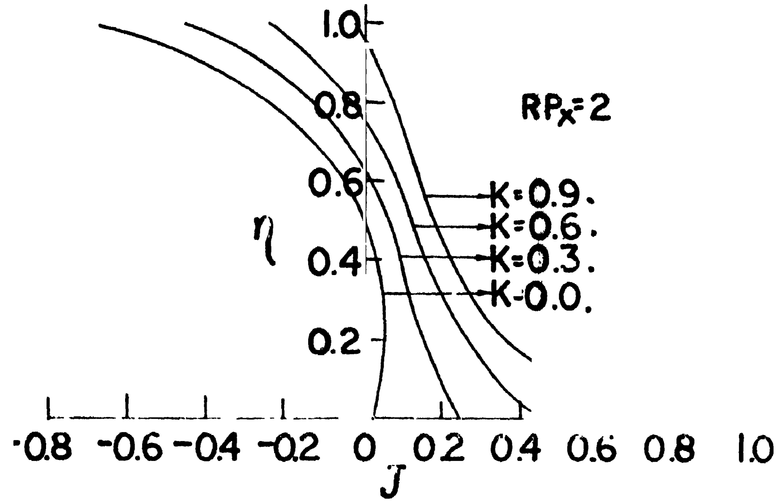

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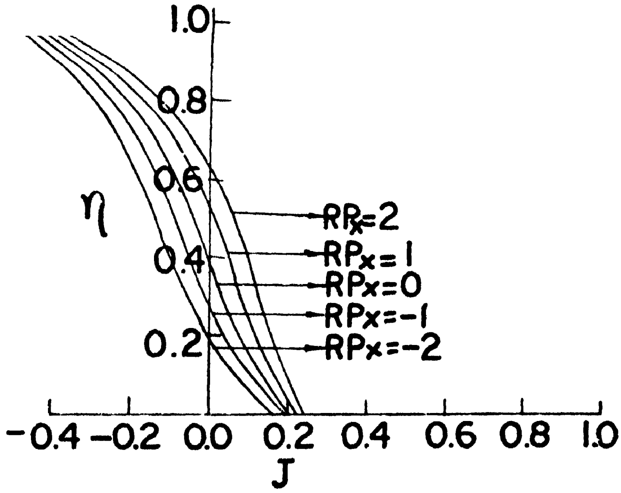


Figure 8.

The numerical values of the velocity gradient at the upper plate are given in table 1.

The shearing stress in non-dimensional form is given by

$$R\tau = \left(\frac{df}{d\eta}\right)_{\eta=0}$$

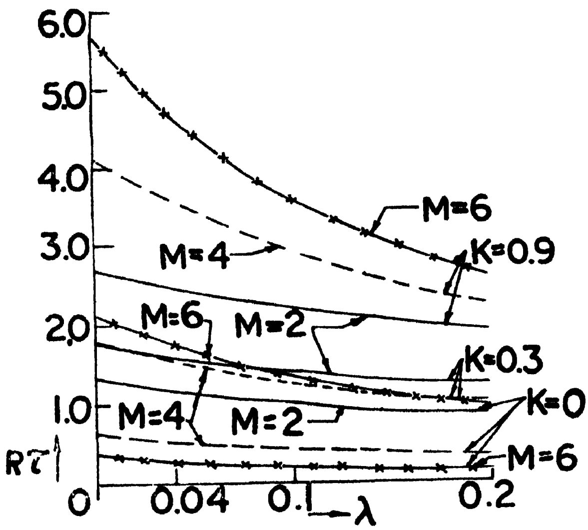
$$\frac{M}{(1+\lambda^2M^2)\sinh M+2\lambda M\cosh M}+\frac{RP_x-M^2K}{M}$$

$$\Big/\left[\frac{1-\cosh M-\lambda M\sinh M}{(1+\lambda^2M^2)\sinh M+2\lambda M\cosh M}\right] \quad (13)$$

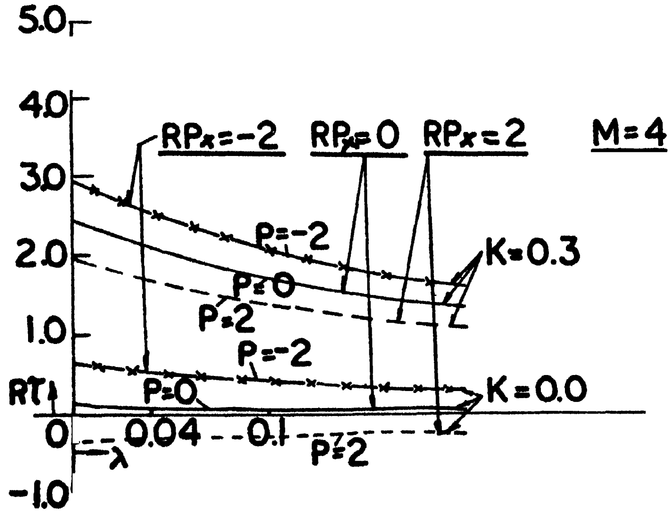
$R\tau$ is plotted on graphs in figures 9 and 10. The flow is now given by,

$$Q = \frac{\left[1+\frac{2(RP_x - M^2K)}{M^2}\right]\left[\frac{\cosh M - 1}{M} + \lambda \sinh M\right]}{(1+\lambda^2 M^2)\sinh M + 2\lambda M \cosh M}$$

Q is plotted in figure 11.



Figuro 9.



Figuro 10.

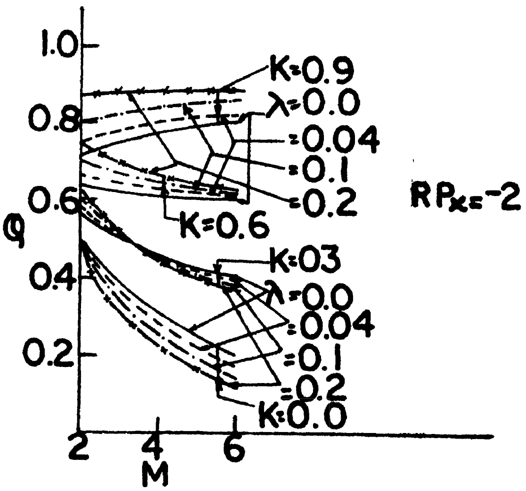


Figure 11.

CONCLUSIONS

It was observed by Soundalgekar (1967) that in no-slip case, for $RP_x = 2$, $K = 0$ and $M = 2$, the velocity is negative over a portion of the channel near the stationary wall indicating the tendency of separation. But for non-zero values of K, the electric field parameter, the velocity for the same values of RP_x and M was found to be positive throughout the channel. Hence separation in case of no-slip condition can be avoided if electric field applied is normal to both the flow field and the magnetic field. In the present case, the velocity profiles, as observed from figures 1 to 4, are positive throughout the channel. Hence separation is found to be absent in the slip-flow regime in the presence of the crossed electric field. In fig. 9, the mass flow Q is plotted agaist the Hartmann number M, for different values of K and $RP_x = -2$. It can be observed from this figure that only for small values of K, an increase in M leads to a decrease in the mass-flow. But for large K, say $K = 0.9$, Q increases with increasing M. In short circuit case ($K = 0$) an increase in λ leads to a decrease in Q whereas in the open circuit case an increase in λ leads to an increase in the mass flows.

From figure 10, we have the following conclusions. R_T increases with increasing K, but decreases with λ. In the short circuit case, it decreases with M but in the open circuit case it increases with M for $RP_x = -2$.

In figure 11, the effects of the varying pressure gradient is shown on R_T. Both in an open or short circuited case, R_T is greater in the presence of an adverse pressure gradient than the one decreasing in the direction of motion.

In table 1, the numerical values of the velocity gradient at the upper moving plate are entered. We have the following observations :

Table 1

Values of $(du/dz)_{z=1}$

RP_x	λ	$k = 0$			$k = 0.3$			$k = 0.6$			$k = 0.9$;		
		$M = 2$	4	6	2	4	6	2	4	6	2	4	6
−2	0.0	1.313	3.521	5.668	0.856	2.364	3.877	0.399	1.207	2.086	−0.058	0.050	0.295
	0.04	1.188	3.032	4.571	0.757	2.030	3.125	0.327	1.028	1.679	−0.104	0.025	0.234
	0.1	1.040	2.510	3.542	0.643	1.675	2.421	0.247	0.840	1.300	−0.150	0.005	0.178
	0.2	0.861	1.950	2.576	0.511	1.297	1.760	0.160	0.644	0.943	−0.189	−0.009	0.127
−1	0.0	1.694	3.762	5.834	1.237	2.605	4.043	0.780	1.448	2.252	0.323	0.291	0.461
	0.04	1.547	3.241	4.705	1.116	2.239	3.259	0.686	1.236	1.813	0.255	0.234	0.367
	0.1	1.370	2.684	3.646	0.974	1.849	2.525	0.577	1.014	1.403	0.181	0.179	0.282
	0.2	1.153	2.086	2.652	0.803	1.433	1.835	0.452	0.780	1.019	0.102	0.127	0.203
0	0.0	2.075	4.003	6.000	1.618	2.846	4.209	1.161	1.689	2.418	0.704	0.532	0.627
	0.04	1.906	3.450	4.839	1.475	2.447	3.393	1.044	1.445	1.947	0.614	0.443	0.501
	0.1	1.701	2.856	3.750	1.304	2.023	2.628	0.908	1.188	1.507	0.511	0.353	0.385
	0.2	1.445	2.222	2.727	1.094	1.569	1.911	0.744	0.916	1.095	0.394	0.263	0.278
1	0.0	2.455	4.244	6.166	1.998	3.087	4.375	1.542	1.930	2.584	1.085	0.773	0.793
	0.04	2.265	3.659	4.973	1.834	2.656	3.527	1.404	1.654	2.081	0.973	0.652	0.635
	0.1	2.031	3.032	3.854	1.635	2.197	2.732	1.238	1.362	1.611	0.842	0.527	0.489
	0.2	1.737	2.358	2.803	1.386	1.705	1.987	1.036	1.052	1.270	0.686	0.399	0.354
2	0.0	2.836	4.485	6.332	2.379	3.328	4.541	1.922	2.171	2.750	1.465	1.014	0.958
	0.04	2.624	3.867	5.106	2.193	2.865	3.661	1.763	1.863	2.215	1.332	0.861	0.769
	0.1	2.363	3.206	3.958	1.965	2.371	2.836	1.569	1.536	1.715	1.172	0.701	0.593
	0.2	2.028	2.494	2.878	1.678	1.841	2.062	1.328	1.188	1.246	0.978	0.535	0.429

1. For the same values of RP_x, λ and M, $\left(\frac{\partial u}{\partial z}\right)_{z=1}$ decreases as K increases. Hence the force to move the plate is less as K increases. Also the force to move the plate in the short-circuit case is more than that in the open circuit case.

2. An increase in λ, with M, RP_x and K fixed, leads to a decrease in the force to move the upper plate

3. The value of $\left(\frac{du}{dz}\right)_{z=1}$ is less in case of $RP_x < 0$ than in case of $RP_x > 0$. Hence for a pressure decreasing in the direction of motion, the force to move the upper plate is less than in case of pressure increasing in the direction of motion.

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